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11/20 Lecture Notes

Theorem: $\det\left(\begin{bmatrix} a_{11} & \cdots & & \\ \vdots & a_{22} & & \star \\ & & a_{33} & \\ O & & & a_{nn} \end{bmatrix}\right) = a_{11} \cdot a_{22} \cdots a_{nn}$ } upper triangular matrix (whatever $\star$)

& $\det\left(\begin{bmatrix} a_{11} & & & O \\ & a_{22} & & \\ & & a_{33} & \\ \star & & & a_{nn} \end{bmatrix}\right) = a_{11} \cdot a_{22} \cdot a_{nn}$ } lower triangular matrix

Ex) $\det\left(\begin{bmatrix} 0&0&1&0 \\ 0&1&0&0 \\ 1&0&0&0 \\ 0&0&0&1 \end{bmatrix}\right) = 0+0+1\left((-1)^8 \det\begin{bmatrix} 0&0&1 \\ 0&1&0 \\ 1&0&0 \end{bmatrix}\right) = 0+0+1\left((-1)^4 \det\begin{bmatrix} 0&1 \\ 1&0 \end{bmatrix}\right) = \det\begin{bmatrix} 0&1 \\ 1&0 \end{bmatrix}$
$= -1$

Theorem: If A & B are nxn matrices then $\det(AB) = \det(A)\det(B)$
↳ Even if $AB \neq BA$, $\det(A)\det(B) = \det(B)\det(A)$

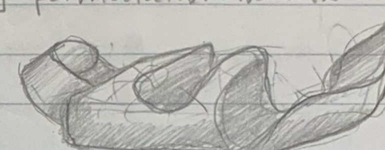
• Row operations are all matrix operations

Swap rows: $\begin{bmatrix} 0&1 \\ 1&0 \end{bmatrix}\begin{bmatrix} a&b \\ c&d \end{bmatrix} = \begin{bmatrix} c&d \\ a&b \end{bmatrix}$ ⇒ where $\begin{bmatrix} a&b \\ c&d \end{bmatrix}$ = permutation matrix

$\det\left(\begin{bmatrix} 0&1 \\ 1&0 \end{bmatrix}\begin{bmatrix} a&b \\ c&d \end{bmatrix}\right) = \det\begin{bmatrix} c&d \\ a&b \end{bmatrix}$

$-1 \det\begin{bmatrix} a&b \\ c&a \end{bmatrix} = \det\begin{bmatrix} c&d \\ a&b \end{bmatrix}$

$\det\begin{bmatrix} a&b \\ c&a \end{bmatrix} = -\det\begin{bmatrix} c&d \\ a&b \end{bmatrix}$



• Let A be a matrix, A' be the matrix after a row swap then $\det A = -\det A'$

can you perform row operations on a matrix before the determinant

Multiply a row by a nonzero constant: $\begin{bmatrix} 1&0&0 \\ 0&7&0 \\ 0&0&1 \end{bmatrix}\begin{bmatrix} a&b&c \\ d&e&f \\ g&h&i \end{bmatrix} = \begin{bmatrix} a&b&c \\ 7d&7e&7f \\ g&h&i \end{bmatrix}$

• Let A be a matrix, A' be the matrix after a row is multiplied by r, then $\det(A) = \frac{1}{r}\det A'$

$\det\left(\begin{bmatrix} 1&0&0 \\ 0&7&0 \\ 0&0&1 \end{bmatrix}\right) \det\begin{bmatrix} a&b&c \\ d&e&f \\ g&h&i \end{bmatrix} = \det\left(\begin{bmatrix} a&b&c \\ 7d&7e&7f \\ g&h&i \end{bmatrix}\right)$

$\det\begin{bmatrix} a&b&c \\ d&e&f \\ g&h&i \end{bmatrix} = \frac{1}{7}\det\left(\begin{bmatrix} a&b&c \\ 7d&7e&7f \\ g&h&i \end{bmatrix}\right)$

Add a multiple of one row to another: $\begin{bmatrix} 1&0&0 \\ 0&1&0 \\ -2&0&1 \end{bmatrix}\begin{bmatrix} a&b&c \\ d&e&f \\ g&h&i \end{bmatrix} = \begin{bmatrix} a&b&c \\ d&e&f \\ g-2a&h-2b&i-2c \end{bmatrix}$

• Let A be a matrix, A' be the matrix after a multiple of one row is added to another row, then $\det A = \det A'$

$\det([\,''\,])\det([\,''\,]) = \det([\,''\,])$

$\det\left(\begin{bmatrix} a&b&c \\ d&e&f \\ g&h&i \end{bmatrix}\right) = \det[\,''\,]$

Ex) what is $\det\begin{bmatrix} 1&2&3&4&5 \\ 2&3&6&0&1 \\ -1&1&2&3&4 \\ 0&2&0&2&0 \\ 1&1&1&-1&3 \end{bmatrix} \Rightarrow = \det\begin{bmatrix} 1&2&3&4&5 \\ 0&-1&0&-8&-9 \\ 0&3&5&7&9 \\ 0&2&0&2&0 \\ 0&-1&-2&5&2 \end{bmatrix} = \det\begin{bmatrix} 1&2&3&4&5 \\ 0&-1&0&-8&-9 \\ 0&0&5&-17&-18 \\ 0&0&0&-14&-18 \\ 0&0&-2&3&7 \end{bmatrix}$

$= \det\begin{bmatrix} 1&2&3&4&5 \\ 0&-1&0&-8&-9 \\ 0&0&5&-17&-18 \\ 0&0&0&-14&-18 \\ 0&0&0&-19/5&-1/5 \end{bmatrix}$ ⇒ use recursive method & first column ⇒ $(1)(-1)(5)\det\begin{bmatrix} -14&-18 \\ -\frac{19}{5}&-\frac{1}{5} \end{bmatrix}$

$= -5\left(\frac{14}{5} - \frac{18\cdot 19}{5}\right) = 14 - 18\cdot 19$

markov chains?

• $\det A^{-1}$ (if A is invertible) $= \frac{1}{\det A}$